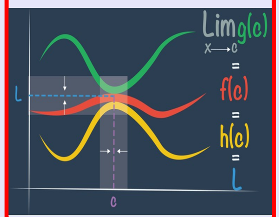
Math 261

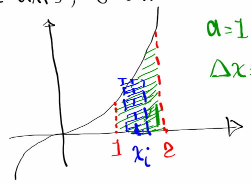
Spring 2023

Lecture 43



Feb 19-8:47 AM

find the area below $f(x)=x^3$, above x-axis, from $x=1$ to $x=2$.



$a=1$, $b=2$

$\Delta x = \frac{b-a}{n} = \frac{1}{n}$

$x_i = a + i\Delta x$

$= 1 + i \cdot \frac{1}{n} = 1 + \frac{i}{n}$

$A_i = f(x_i) \cdot \Delta x = \left(1+\frac{i}{n}\right)^3 \cdot \frac{1}{n}$

Recall

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$\left(1+\frac{i}{n}\right)^3 = 1 + \frac{3i}{n} + \frac{3i^2}{n^2} + \frac{i^3}{n^3}$

$A_i = \left(1+\frac{i}{n}\right)^3 \cdot \frac{1}{n} = \frac{1}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}$

$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \left[\frac{1}{n} + \frac{3i}{n^2} + \frac{3i^2}{n^3} + \frac{i^3}{n^4}\right]$

$= \sum_{i=1}^{n} \frac{1}{n} + \sum_{i=1}^{n} \frac{3i}{n^2} + \sum_{i=1}^{n} \frac{3i^2}{n^3} + \sum_{i=1}^{n} \frac{i^3}{n^4}$

$= \frac{1}{n}\sum_{i=1}^{n} 1 + \frac{3}{n^2}\sum_{i=1}^{n} i + \frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n^4}\sum_{i=1}^{n} i^3$

May 1-9:50 AM

$$= \frac{1}{n}\sum_{i=1}^{n} 1 + \frac{3}{n^2}\sum_{i=1}^{n} i + \frac{3}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n^4}\sum_{i=1}^{n} i^3$$

Recall from Pre-Calc.

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6},$$

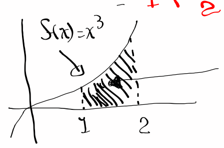
$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{1}{n}\cdot n + \frac{3}{n^2}\cdot\frac{n(n+1)}{2} + \frac{3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{1}{n^4}\left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{n}{n} + \frac{3n^2+\text{Junk}}{2n^2} + \frac{6n^3+\text{Junk}}{6n^3} + \frac{n^4+\text{Junk}}{4n^4}$$

$$\lim_{n\to\infty} A_n = \lim_{n\to\infty}\left[\cancel{1}^{\,1} + \cancel{\frac{3n^2+\text{Junk}}{2n^2}}^{\,3/2} + \cancel{\frac{6n^3+\text{Junk}}{6n^3}}^{\,1} + \cancel{\frac{n^4+\text{Junk}}{4n^4}}^{\,1/2}\right]$$

$$= 1 + \frac{3}{2} + 1 + \frac{1}{4} = 1 + 1.5 + 1 + .25$$



Area = $\boxed{3.75}$

May 2-8:53 AM

Introduction To integration

$$\int f(x)\,dx = F(x) + C$$

Integral

$$\int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int 2x\,dx = x^2 + C$$

Find $\int x^3 dx = \dfrac{x^{3+1}}{3+1} + C = \boxed{\dfrac{1}{4}x^4 + C}$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

if $n \neq -1$

To verify

$$\frac{d}{dx}\left[\frac{1}{4}x^4 + C\right]$$

$$= \frac{1}{\cancel{4}}\cdot\cancel{4}x^3 + 0 = \boxed{x^3}$$

May 2-9:06 AM

find $\int \sqrt{x}\, dx = \int x^{1/2}\, dx$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{3} x^{3/2} + C}$$

To verify

$$\frac{d}{dx}\left[\frac{2}{3} x^{\frac{3}{2}} + C\right] = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} \cdot x^{\frac{3}{2}-1} + 0$$

$$= x^{\frac{1}{2}} = \boxed{\sqrt{x}}$$

May 2-9:13 AM

$\int f'(x)\, dx = f(x) + C$ Indefinite Integral

$\int_a^b f'(x)\, dx = f(x) \Big|_a^b = f(b) - f(a)$

Definite integral

$$\int_1^2 x^3\, dx = \frac{x^{3+1}}{3+1} \Big|_1^2 = \frac{1}{4} x^4 \Big|_1^2$$

$$= \frac{1}{4}\left[2^4 - 1^4\right] = \frac{1}{4} \cdot 15 = \boxed{3.75}$$

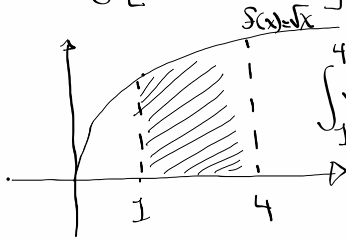
May 2-9:16 AM

Evaluate $\int_1^4 \sqrt{x}\,dx$

$$\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_1^4$$

$$= \frac{x^{3/2}}{3/2}\Bigg|_1^4 = \frac{2}{3}x^{3/2}\Bigg|_1^4 = \frac{2}{3}\cdot x\sqrt{x}\Bigg|_1^4$$

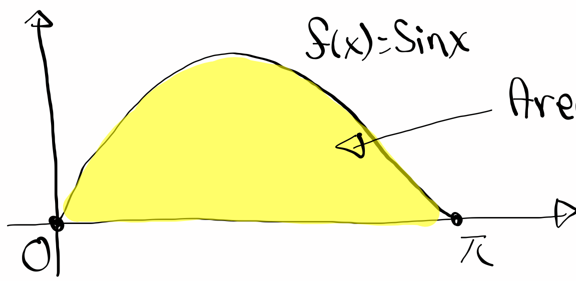
$$= \frac{2}{3}\left[4\sqrt{4} - 1\sqrt{1}\right] = \frac{2}{3}\cdot\left[8-1\right] = \frac{2}{3}\cdot 7 = \boxed{\frac{14}{3}}$$



$$\int_1^4 \sqrt{x}\,dx = \frac{14}{3}$$

May 2-9:20 AM

Find the area below $f(x) = \sin x$, above x-axis from $x=0$ to $x=\pi$.



$$\text{Area} = \int_0^\pi \sin x\,dx$$

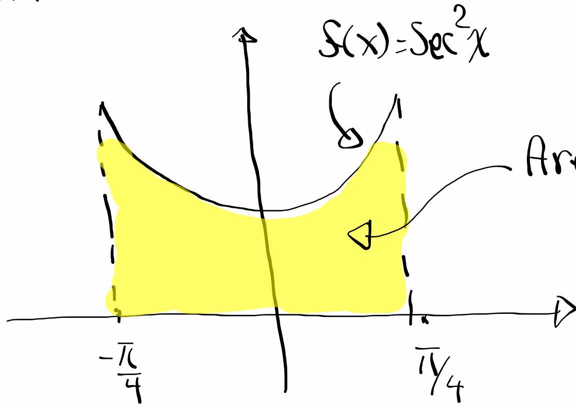
$$= -\cos x\Big|_0^\pi$$

$$= -\left[\overset{-1}{\cos\pi} - \overset{1}{\cos 0}\right]$$

$$= -\left[-1 - 1\right] = \boxed{2}$$

May 2-9:25 AM

Find the shaded area below:



$$\text{Area} = \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx$$

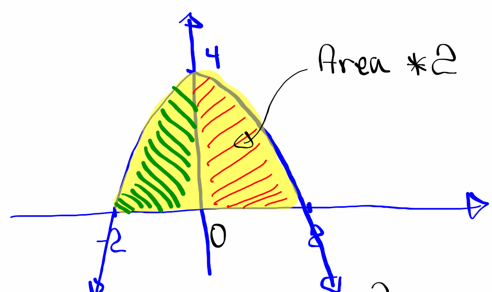
$$= \tan x \Big|_{-\pi/4}^{\pi/4}$$

$$= \tan\frac{\pi}{4} - \tan\left(\frac{-\pi}{4}\right) = \tan\frac{\pi}{4} + \tan\frac{\pi}{4} = 1 + 1 = \boxed{2}$$

Recall $\tan(-\alpha) = -\tan\alpha$

May 2-9:31 AM

Find the area enclosed by $f(x)=4-x^2$ and the x-axis.



$$A = 2\int_0^2 (4-x^2)\,dx = 2\left[4x - \frac{x^3}{3}\right]\Bigg|_0^2$$

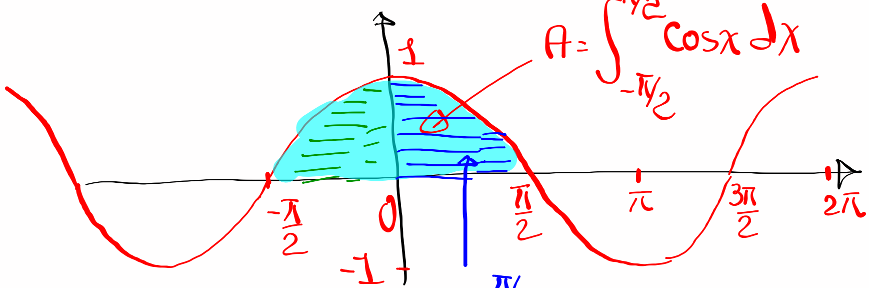
$$= 2\left[4(2) - \frac{2^3}{3} - 0\right] = 2\left[8 - \frac{8}{3}\right]$$

$$= 2 \cdot \frac{16}{3} = \boxed{\frac{32}{3}}$$

May 2-9:35 AM

Find the area enclosed by x-axis, $f(x) = \cos x$, from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

$$A = \int_{-\pi/2}^{\pi/2} \cos x \, dx$$



$$\text{Area} = 2\int_0^{\pi/2} \cos x \, dx$$

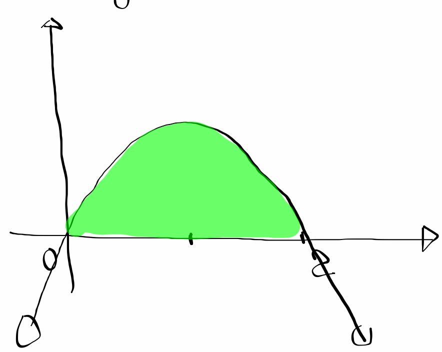
$$= 2 \cdot \sin x \Big|_0^{\pi/2} = 2\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= 2(1) = \boxed{2}$$

May 2-9:42 AM

Find the area enclosed by x-axis and $f(x) = 2x - x^2$.

$$= x(2-x)$$



$$A = \int_0^2 (2x - x^2)\,dx$$

$$= \left[x^2 - \frac{x^3}{3}\right]\Big|_0^2 = \left(2^2 - \frac{2^3}{3}\right) - \left(0^2 - \frac{0^3}{3}\right)$$

$$= 4 - \frac{8}{3} - 0$$

$$= \boxed{\frac{4}{3}}$$

May 2-9:48 AM